

TABLE I. Combinations of elastic constants governing pure mode propagation in cubic crystals.

Propagation direction	Particle motion	Type	Elastic constants
[100]	[100]	Longitudinal	C_{11}
[110]	[110]	Shear	$(C_{11} - C_{12})/2$
[100]	\perp to [100]	Shear	C_{44}
[110]	[110]	Longitudinal	$(C_{11} + C_{12} + 2C_{44})/2$
[110]	[001]	Shear	C_{44}
[111]	[111]	Longitudinal	$(C_{11} + 2C_{12} + 4C_{44})/3$
[111]	\perp to [111]	Shear	$(C_{11} - C_{12} + C_{44})/3$

and Ruoff⁸ assumed a linear variation of Δ vs P as described below. We note that

$$\rho = \rho_0 \lambda^3. \quad (14)$$

Beginning with Eq. (9) and substituting for B^S and ρ from (12) and (14), we obtain

$$\Delta(P) = \frac{T\beta^2 \{ [L_1^2(P=0)/\tau_1^2(P)] - \frac{4}{3} [L_2^2(P=0)/\tau_2^2(P)] \}}{\lambda^2 C_P}. \quad (15)$$

Next express $\Delta(P)$ as a power-series expression.

$$\Delta(P) = \Delta_0 + \Delta_0' P + \Delta_0'' (P^2/2) + \dots \quad (16)$$

The logarithmic derivative of Δ as given by (15) yields

$$\Delta^{-1} \left(\frac{\partial \Delta}{\partial P} \right)_T = \frac{2}{\beta} \left(\frac{\partial \beta}{\partial P} \right)_T - \frac{2}{\lambda} \left(\frac{\partial \lambda}{\partial P} \right)_T - \left(\frac{L_1^2(P=0)}{\tau_1^2(P)} - \frac{4}{3} \frac{L_2^2(P=0)}{\tau_2^2(P)} \right)^{-1} \frac{\partial}{\partial P} \times \left(\frac{L_1^2(P=0)}{\tau_1^2(P)} - \frac{4}{3} \frac{L_2^2(P=0)}{\tau_2^2(P)} \right)_T - C_P^{-1} \left(\frac{\partial C_P}{\partial P} \right)_T. \quad (17)$$

We now proceed to evaluate $(\partial \Delta / \partial P)_T$ as $P \rightarrow 0$, i.e., Δ_0' . From Eq. (6) we have

$$\lim_{P \rightarrow 0} - (2/\lambda) (\partial \lambda / \partial P)_T = -2/3 B_0 T. \quad (18)$$

TABLE II. Internal consistency of the adiabatic elastic constant values (in units of 10^{11} dyn/cm²) of RbCl, RbBr, and RbI as a function of temperature ($^{\circ}$ K) at 1 atm.

Temperature	C_{11}		C_{12}		C_{44}	
	(1) ^a	(2) ^b	(1) ^a	(2) ^b	(1) ^a	(2) ^b
III RbI						
300	2.5570	2.5561	0.3766	0.3815	0.2772	0.2777
260	2.6496	2.6519	0.3675	0.3641	0.2792	0.2799
220	2.7490	2.7474	0.3514	0.3506	0.2818	0.2810
180	2.8504	2.8502	0.3354	0.3352	0.2840	0.2844
I RbCl						
300	3.6213	3.6242	0.6154	0.6124	0.4668	0.4678
260	3.7486	3.7519	0.6070	0.6034	0.4713	0.4714
220	3.8756	3.8831	0.5934	0.5899	0.4754	0.4754
180	4.0011	4.0071	0.5753	0.5745	0.4789	0.4794
II RbBr						
300	3.1513	3.1522	0.4995	0.5000	0.3798	0.3801
260	3.2570	3.2624	0.4825	0.4860	0.3831	0.3834
220	3.3691	3.3727	0.4714	0.4731	0.3863	0.3866
180	3.4882	3.4882	0.4587	0.4593	0.3898	0.3900

^a From the experimental runs at higher pressures as a function of temperature.

^b From the experimental runs at one atmosphere as a function of temperature.

TABLE III. Adiabatic elastic constants (in units of 10^{11} dyn/cm²) and density (in units of g/cm³) of RbCl as a function of temperature (°K) as obtained in present work (PW) and those obtained by Marshall, Pederson, and Dorris (MPD).

Temp.	C_{11}		C_{12}		C_{44}		Bulk modulus		Density		Δ
	PW	MPD	PW	MPD	PW	MPD	PW	MPD	PW	MPD	
300	3.6242	3.653	0.6124	0.645	0.4678	0.478	1.616	1.647	2.7969	2.7972	0.0533
280	3.6894		0.6053		0.4695		1.633		2.8033		0.0502
260	3.7519		0.6034		0.4714		1.653		2.8097		0.0471
240	3.8189	3.840	0.5959	0.666	0.4735	0.482	1.670	1.724	2.8160	2.8163	0.0437
220	3.8831		0.5899		0.4754		1.688		2.8223		0.0403
200	3.9448		0.5812		0.4775		1.702		2.8285		0.0367
180	4.0071	4.025	0.5745	0.676	0.4794	0.486	1.719	1.792	2.8346	2.8345	0.0331
160	4.0681		0.5630		0.4813		1.731		2.8404		0.0293
140	4.1194		0.5569		0.4831		1.744		2.8458		0.0254
120	4.1792	4.206	0.5516	0.676	0.4848	0.490	1.761	1.852	2.8504	2.8507	0.0214
100	4.2394	4.266	0.5484	0.676	0.4858	0.491	1.779	1.873	2.8538		0.0172

We also have the following thermodynamic relation:

$$(\partial\beta/\partial P)_T = -[(\partial/\partial T)(1/B^T)]_P. \quad (19)$$

This is true at all pressures, but in the limit we have

$$\lim_{P \rightarrow 0} (\partial\beta/\partial P)_T = (1/B_0^T) (\partial B^T/\partial T)_{P=0}. \quad (20)$$

Likewise

$$\begin{aligned} (\partial C_P/\partial P)_T &= -T[(\partial^2/\partial T^2)(1/\rho)]_P \\ &= -(T/\rho)[(\partial\beta/\partial T)_P + \beta^2]. \end{aligned} \quad (21)$$

Again at the limit we have

$$\lim_{P \rightarrow 0} (\partial C_P/\partial P)_T = -(T/\rho_0)[(\partial\beta/\partial T)_{P=0} + \beta_0^2]. \quad (22)$$

Therefore *all* the terms on the right-hand side of Eq. (17) are known in the limit as $P \rightarrow 0$ if we have sufficient thermal expansion and specific-heat data as a function of temperature at zero pressure to combine with the direct experimental values which we measure and which give the second and third terms. Thus we can evaluate Δ_0' . Hence we have

$$\Delta(P) = \Delta_0 + \Delta_0' P$$

TABLE IV. Temperature derivatives of the adiabatic elastic constants (in units of 10^7 dyn/cm² °K) of RbCl at room temperature.

	Temperature (°K)	C_{11}	C_{12}	C_{44}	Bulk modulus
Present work	300	-30.80	3.40	-0.934	-8.00
Hausuhl	295	-32.12	3.46	-1.094	-8.40
Marshall <i>et al.</i>	300	-31.10	-3.50	-0.667	-12.7

which is then substituted into Eq. (13). Thus we can evaluate λ from (13) and then we get all the pertinent elastic constants. (Note that one does *not* have to have approximate values of $(\partial B_0^S/\partial P)_T$ to carry out this analysis.) Combinations of elastic constants for pure-mode propagation in cubic crystals are given in Table I. In the following sections the elastic constants of RbCl, RbBr, and RbI as a function first of temperature at one atmosphere and then of pressure at several temperatures are discussed. The values presented are (i) based on the travel times evaluated at the resonant frequency of quartz at a pressure P and temperature T . (ii) They are the least-square estimates obtained from four $\rho(P)v(P)$ values, and (iii) they are internally self-consistent. As an example of self-consistency we present the values of the elastic constants of RbCl, RbBr, and RbI as a function of temperature at one atmosphere in Table II. The two values of each constant at a temperature are obtained from two independent experimental runs as indicated underneath Table II. The agreement between these pairs of values are within the range of experimental errors. These errors are estimated to be 0.2%, 0.6%, and 0.4% for C_{11} , C_{12} , and C_{44} , respectively, for each of the rubidium halides. The errors associated with these values of the elastic constants incorporate not only the usual errors of measurements like length of the specimens, absolute null frequency and its reproducibility, and misorientation, but also the statistical or random errors due to replication. The associated errors are thus larger in magnitude than the precision with which each experimental run could be executed. Where some pertinent ancillary data were not available, specific assumptions had to be made. These assumptions are explicitly described in the separate sections dealing with the halides to which they specifically